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THE ROUNDNESS OF POLYHEDRA

Pieter Huybers

Building egineer, former associate professor at the Delft University of Technology, The Netherlands.
Address: Oosterlee 16, 2678 AZ, De Lier, The Netherlands.
Fields of interest: Structural morphology, polyhedra, geometry, space structures, geodesic domes, construction of soccer balls.
Awards: Tsuboi award of IASS in 2000.
Websites: www.pieterhuybers.nl and www.hyperball.nl
Publications: over 250 on geometry, load bearing plastics structures, roundwood space structures, dome subdivisions.

Abstract: *The roundness of convex uniform polyhedra can sometimes be of utmost importance. One of the Archimedean solids, the truncated icosahedron, is since the eighties in use for the construction of the outer skin of soccer balls and in this case the roundness of its form is essential and is sometimes of great economic value. This form is not ideal, particularly because the faces of this solid differ very much in area (about 51%) and have different distances from the systems center. It is an open question whether any of the other 12 semi-regular solids may eventually have characteristics that make it more suitable for this purpose. As they all consist of two or three kinds of different polygons and as these also have different distances from the center, a general approach has been worked out to improve the roundness of polyhedral solids.*

Keywords: Geometry, polyhedron, isohedron, structures, space frames, domes.

1. DEFINITION OF ROUNDNESS

The roundness of a polyhedron can be defined by the following conventions:
1. All faces touch the same inscribed sphere: have the same distance from the systems center.
2. All corners lie on one circumscribed sphere and it is convex.
3. The radius of the inscribed sphere is closest to that of the circumscribed sphere.
4. It has the smallest deficient angle: the missing part of the full 360^{o}, if all faces that meet at a corner are put together.

2. PLATONIC AND ARCHIMEDEAN ISOHEDRA

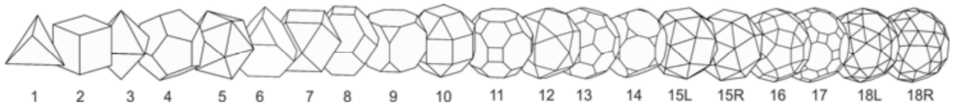


Figure 1: Platonic solids (1 to 5) and Archimedean solids (6 to 20)

The convex regular or Platonic polyhedra have only one kind of faces, all equal regular polygons. All faces touch an inscribed sphere with the radius R_i. So they are isohedra by origin. The vertex congruent semi-regular or Archimedean polyhedra have two or three kinds of faces in the form of regular polygons with 3, 4, 5, 6, 8 or 10 sides. We indicate here all solids by a serial number as P1 to P18. P15 and P18 have a left-handed and a right-handed version. All corners of the polygons lie on one circumscribed sphere with the radius R_e, but if more than one kind occurs, they do not have the same distance from the systems centre and thus lie on two or three inscribed spheres. A general rule in a given sphere is: the larger the polygon the closer it is to the centre of the system. The deficient angle is: 360^o minus the sum of the meeting face angles. The smaller this angle, the closer it approximates the plane surface and thus is averagely smoother.

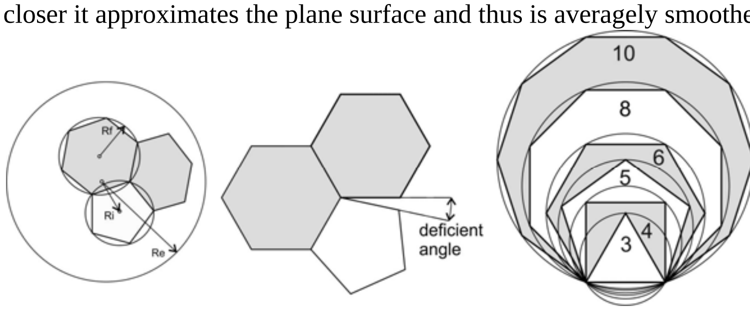


Figure 2: The faces are polygonal and have different circumcircles R_f. The Archimedean solids are found by cutting off parts either from the octahedron or from the icosahedron. So we discern these two main groups.

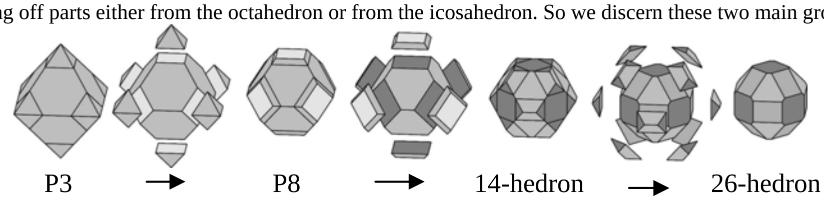


Figure 3: The octahedron group. If the truncation process is done at the same distance as that of the triangles in the basic octahedron, we get isohedra. We call these after their numbers of faces. P7, P8 en P9 have the 14-hedron in common, P10 and P11 lead to the 26-hedron.

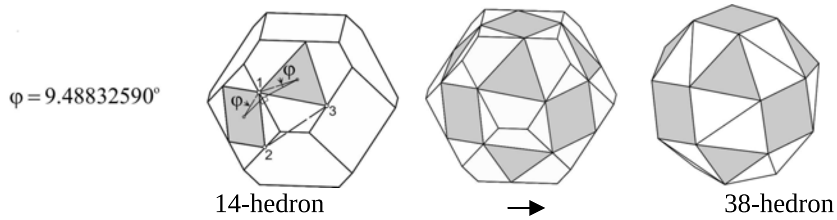


Figure 4: The 38-hedron in its turn is derived from the 14-hedron. In this figure is shown that this is done by rotating a triangle and a square lying in its faces over the angle **φ**, while keeping their corners sliding along its sidelines, and then connecting points 2 and 3 until the ratio of the sides 1-2, 1-3 and 2-3 has reached a certain value. The new formed plane 1-2-3 has all its points at the distance R_i and hence as a whole it is isodistant with the others.

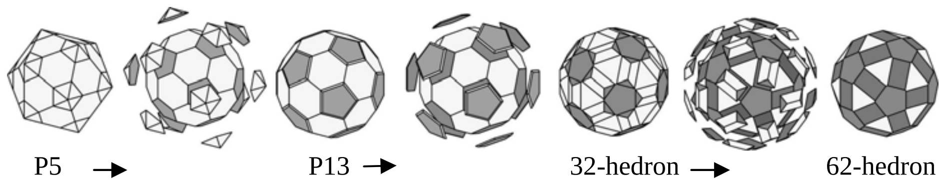


Figure 5: The icosahedral group, P12, P13 and P14, has the 32-hedron in common; P16 and P17 lead to the 62-hedron.

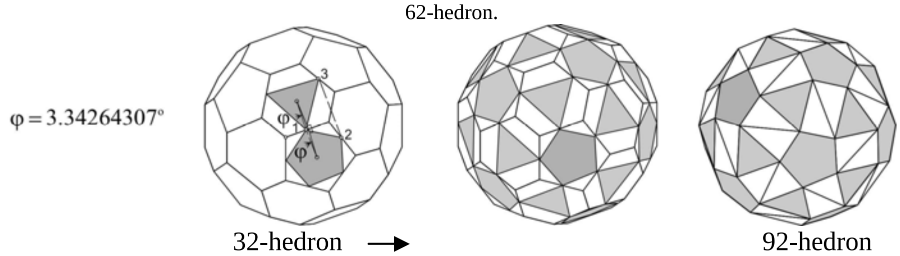


Figure 6: The 92-hedron is found in a similar way as the 38-hedron, but is now derived from the 32-hedron.

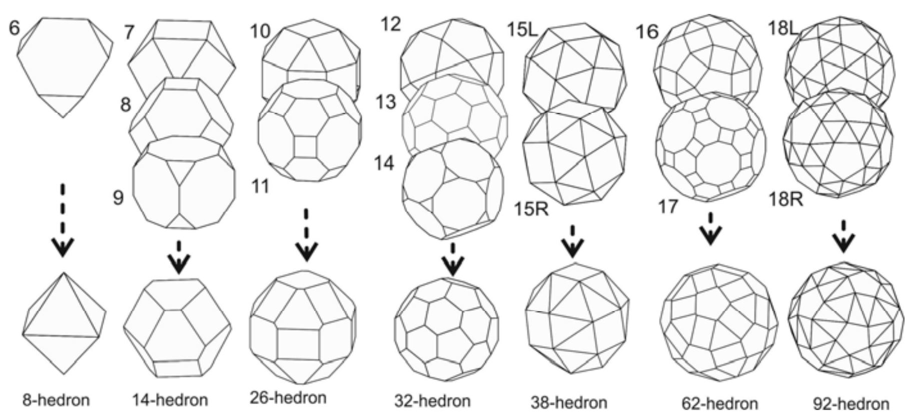
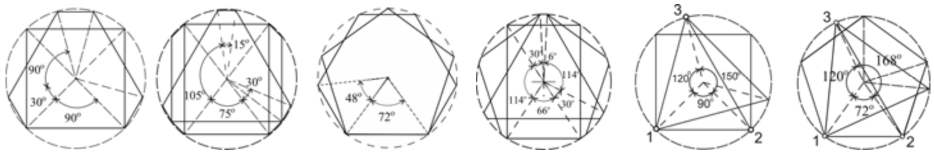


Figure 7: Review of the Archimedean isohedra, indicated with their numbers of faces.



14-hedron 26-hedron 32-hedron 62-hedron 38-hedron 92-hedron

Figure 8: All faces in any specific isohedron lie on one circumscribed circle R_f.

3. SOCCER BALLS

As indicated before, the construction of the outer skin of soccer balls is usually based on that of P13, consisting of 20 hexagons and 12 pentagons. But this solution is not optimal. Some of the isohedra have characteristics that make them more suitable for this application. A few of these have been investigated. This has lead to a ball design, called Hyperball. This is actually based on the 62-hedron. Together with the 92-hedron it has the smallest difference between R_e and R_i (5,6%). But in the end the 62-hedron has been preferred, as the other is much more difficult to construct. In order to reduce the seam length, for the Hyperball the rectangular panels were subdivided and added to adjacent panels. So in the end a composition is obtained, again consisting of 12 pentagon-like and 20 hexagon-like panels, almost as in the traditional P13. All panels of this new ball though have the same R_i, have similar inscribed circles, have almost the same surface area (5.2% difference, instead of 51% in the P13-ball), have the smallest deficient angle of all (only 6^o) and thus are closest to the ideal circumscribed sphere.

92-hedron 26-hedron 62-hedron → Hyperball

Figure 9: Three soccer ball forms based on isohedra. Hyperball is presently being produced in larger numbers.

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